Algebraic sets with fully characteristic radicals

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Let G be a group and S be a system of group equations with coefficients in G. We denote by $\mathrm{Rad}_G(S)$ the set of all group equations which are logical consequences of S in G. In general, one can not give a deductive description of $\mathrm{Rad}_G(S)$, because it depends on the axiomatizablity of the prevariety generated by G. In this direction, any good description of the radicals is important from the algebraic geometric point of view.

In this talk, we give a necessary and sufficient condition for $\mathrm{Rad}_G(S)$ to be fully characteristic (invariant under all endomorphisms). We apply our main result to obtain connections between radicals, identities, coordinate algebras and relatively free groups. Although most of the results can be formulate in the general frame of arbitrary algebraic structures, we mainly focus on groups in what follows. As a summary, we give here some results in the case of coefficient free algebraic geometry of groups.

Let $E \subseteq G^n$ be an algebraic set (with no coefficients). Then the radical $\mathrm{Rad}(E)$ is a fully characteristic (equivalently verbal) subgroup of the free group F_n, if and only if, there exists a family $\{K_i\}$ of n-generator subgroups of G such that $E = \bigcup_i K_i^n$. As a result, we will show that if $\mathrm{Rad}_G(S)$ is a verbal subgroup of F_n, then there exists a family $\mathfrak{X}$ of n-generator subgroups of G such that $\mathrm{Rad}_G(S)$ is exactly the set of all group identities valid in $\mathfrak{X}$. We also see that under this conditions, there exists a variety $\mathbf{W}$ of groups, such that the n-generator relatively free group in $\mathbf{W}$ is the coordinate group of S. We will prove also that if G is a nilpotent group of class at most n and $E \subseteq G^n$ is an algebraic set, then $\mathrm{Rad}(E)$ is a characteristic subgroup of F_n, if and only if $E = \bigcup_i K_i^n$ for some family $\{K_i\}$ of n-generator subgroups of G.

References

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