Triangle-Free Commuting Conjugacy Class Graphs

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There are many ways a graph is associated with conjugacy classes of a group. In 2009, Herzog, Longobardi and Maj [1] introduced the *commuting conjugacy class graph* $\Gamma(G)$ of G associated with the non-central conjugacy classes of G. The vertices of $\Gamma(G)$ are the non-central conjugacy classes of G and two distinct vertices C and D are adjacent whenever there exist two elements $x \in C$ and $y \in D$ such that $xy = yx$. They prove, in particular that for non-abelian periodic groups G, $\Gamma(G)$ is an empty graph if and only if G is isomorphic to one of the groups S_3, D_8 or Q_8.

The aim of this article is to classify all finite groups G with a triangle-free commuting conjugacy class graph. We will state the structure of all groups G with $\Gamma(G)$ is a triangle-free whenever G has odd or even order, G is non-abelian soluble or centerless non-soluble. For instance, we prove the following :

Theorem. *Let G be a finite group whose commuting conjugacy class graph $\Gamma(G)$ is a triangle-free.*

(i) *If G is a group of odd order, then $|G| = 21$ or 27 .*

(ii) *Suppose G is a group of even order which is not a 2-group. If $Z(G) \neq 1$ then G is isomorphic to D_{12} or $T_{12} = \langle a, b \big| a^4 = b^3 = 1, b^a = a^{-1} \rangle$.*

(iii) *If G is a centerless non-soluble group, then G is isomorphic to one of the groups $PSL(2, q)$ ($q \in \{4, 7, 9\}$), $PSL(3, 4)$ or* SmallGroup$(960, 11357)$.

(iv) *If G is a non-abelian soluble group with $Z(G) = 1$, then G is isomorphic to one of the groups: S_3, D_{10}, A_4, S_4,* SmallGroup$(72, 41)$, SmallGroup$(192, 1023)$ *or* SmallGroup$(192, 1025)$.

Note that the nth group of order m in the GAP small groups library is denote by SmallGroup(m, n).

References

[1] M. Herzog, P. Longobardi and M. Maj, On a commutinggGraph on conjugacy classes of groups, *Comm. Algebra* **37**(10) (2009), 3369–3387.

[2] A. Mohammadian, A. Erfanian, M. Farrokhi D.G. and B. Wilkens, Triangle-free commuting conjugacy classes graphs, *Journal of Group Theory*, (2016) to appear.

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