Universal groups of intermediate growth and their invariant random subgroups

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Invariant random subgroup (abbreviated IRS) is a convenient term that stands for a probability measure on the space of subgroups of a group, invariant under the action of the group by conjugation. In the case of a countable group G the space $S(G)$ of subgroups of G is supplied with the topology induced from the Tychonoff topology on $\{0,1\}^G$ where a subgroup $H \leq G$ is identified with its characteristic function $\chi_H(g) = 1$ if $g \in H$ and 0 otherwise. The delta mass corresponding to a normal subgroup is a trivial example of an IRS, as well as the average over a finite orbit of delta masses associated with subgroups in a finite conjugacy class. Hence, we are rather interested in continuous invariant probability measures on $S(G)$. Clearly, such a measure does not necessarily exist, for example if the group only has countably many subgroups. Given a countable group G, a basic question is whether a continuous IRS exists. Ultimately one wants to describe the structure of the simplex of invariant probability measures of the topological dynamical system $(Inn(G), S(G))$ where $Inn(G)$ is the group of inner automorphisms of G acting on $S(G)$. Of particular interest are ergodic measures, i.e., the extremal points in the simplex.

A very fruitful idea in the subject belongs to Anatoly Vershik who introduced the notion of a totally non free action of a locally compact group G on a space X with invariant measure μ, i.e., an action with the property that different points $x \in X$ have different stabilizers $St_G(x)$ μ-almost surely. Then the map $St : X \to S(G)$ defined by $x \mapsto St_G(x)$ is injective μ-almost surely and the image of μ under this map is the law of an IRS on G which is continuous and ergodic whenever μ is.

Recall that, given a finitely generated group G with a system of generators S, one can consider its growth function $\gamma(n) = \gamma_{(G,S)}(n)$ which counts the number of elements of length at most n. The growth type of this function when $n \to \infty$ does not depend on the generating set S and can be polynomial, exponential or intermediate. The question of existence of groups of intermediate growth was raised by Milnor and was answered by Grigrochuk in [2]. The main construction associates with every sequence $\omega \in \Omega = \{0,1,2\}^{\mathbb{N}}$ a group G_ω generated by four involutions $a_\omega, b_\omega, c_\omega, d_\omega$ and if ω is not an eventually constant sequence, then G_ω has intermediate growth. Moreover, it was also observed in [2] that the groups G_ω fall into

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the class of just-infinite branch groups. A group is just infinite if it is infinite but every proper quotient is finite. A group is branch if it has a faithful level transitive action on a spherically homogeneous rooted tree with the property that rigid stabilizers of the levels of the tree are of finite index. Since the groups G_ω are just-infinite, they only have countably many quotients. This raised the question of existence of groups of intermediate growth having $2^{\aleph_0}$ quotients. Our main theorem is:

Theorem. *There exists a finitely generated group of intermediate growth with $2^{\aleph_0}$ distinct continuous ergodic invariant random subgroups.*

The main idea is to take a suitable subset $\Lambda \subset \Omega$ of cardinality $2^{\aleph_0}$ and consider the group U_Λ (which we call the universal group associated to this family) defined as the quotient of the free group F_4 by a normal subgroup N which is the intersection of normal subgroups $N_\omega, \omega \in \Lambda$ where $G_\omega = F_4/N_\omega$. In this paper we explore this idea further by using IRS on G_ω and lift them to U_Λ deducing the main result.

References

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[2] R. I. Grigorchuk. Degrees of growth of finitely generated groups and the theory of invariant means. *Izv. Akad. Nauk SSSR Ser. Mat.*, 48(5):939–985, 1984.